Electromagnetic Induction and Alternating Current

Fill in the Blanks

Q.1. A uniformly wound solen oidal coil of self inductance 1.8 × 10^{-4} henry and resistance 6 ohm is broken up into two identical coils. These identical coils are then connected in parallel across a 15-volt battery of negligible resistance. The time constant for the current in the circuit is .... seconds and the steady state current through the battery is .... amperes. (1989 - 2 Marks)

Ans. 0.3×10^{-4} sec, 10 A

Solution. The coil is broken into two identical coils.

$$L_{eq} = \frac{L/2 \times L/2}{L/2 + L/2} = \frac{L}{4} = 0.45 \times 10^{-4}\,\text{H},$$

$$R_{eq} = \frac{R/2 \times R/2}{R/2 + R/2} = \frac{R}{4} = 1.5\,\Omega$$

$$\text{Time constant} = \frac{L_{eq}}{R_{eq}} = \frac{0.45 \times 10^{-4}}{1.5} = 0.3 \times 10^{-4}\,\text{s}$$

$$\text{Steady current } I = \frac{E}{R_{eq}} = \frac{15}{1.5} = 10\,\text{A}.$$

Q.2. In a straight conducting wire, a constant current is flowing from left to right due to a source of emf. When the source is switched off, the direction of the induced current in the wire will .... (1993 - 1 Marks)

Ans. Left to right

Solution. NOTE : As the source is switched off, the current decreases to zero. The induced current opposes the cause as per Lenz's law. Therefore, the induced current will direct from left to right.

True/ False

Q.1. An e.m.f. can be induced between the two ends of a straight copper wire when it is moved through a uniform magnetic field. (1980)

Ans. T

Solution. True. A copper wire consists of billions and billions of free electrons. When the wire is at rest, the average velocity of each electron is zero. But when the wire is in motion, the electrons have a net velocity in the direction of motion.

NOTE : A charged particle moving in a magnetic field experiences a force given by $\vec{F} = q(\vec{v} \times \vec{B})$.

Here also each electron experiences a force and therefore, electrons will move towards one end creating an emf between the two ends of a straight copper wire.

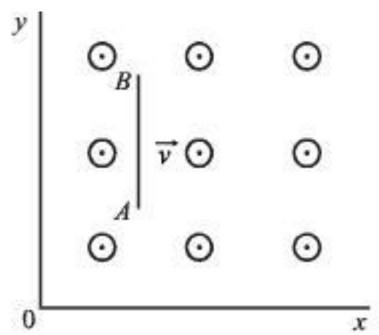
Q.2. A coil of metal wire is kept stationary in a non-uniform magnetic field. An e.m.f. is induced in the coil. (1986 - 3 Marks)

Ans. F

Solution.NOTE : For induced emf to develop in a coil, the magnetic flux through it must change.

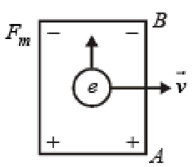
But in this case the number of magnetic lines of force through the coil is not changing. Therefore the statement is false.

Q.3. A conducting rod AB moves parallel to the x-axis (see Fig.) in a uniform magnetic field pointing in the positive z-direction. The end A of the rod gets positively charged. (1987 - 2 Marks)



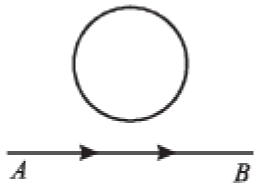
Ans. T

Solution. NOTE : When conduction rod AB moves parallel to x-axis in a uniform magnetic field pointing in the positive z-direction, then according to Fleming's left hand rule, the electrons will experience a force towards B. Hence, the end A will become positive.



Subjective Questions Part -1

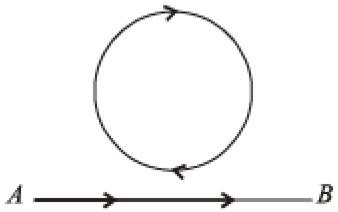
Q.1. A current from A to B is increasing in magnitude. What is the direction of induced current, if any, in the loop as shown in the figure? (1979)



Ans. Clockwise

Solution. The magnetic lines of force due to current flowing in wire AB is shown.

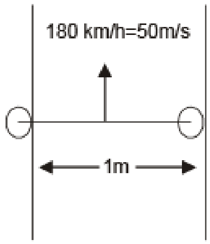
NOTE : As the current increases, the number of magnetic lines of force passing through the loop increases in the outward direction. To oppose this change, the current will flow in the clockwise direction.



Q.2. The two rails of a railway track, insulated from each other and the ground, are connected to a milli voltmeter. What is the reading of the milli voltmeter when a train travels at a speed of 180 km/hour along the track, given that the vertical component of earth's magnetic field is 0.2×10^{-4} weber/m^2 & the rails are separated by 1 meter? (1981- 4 Marks)

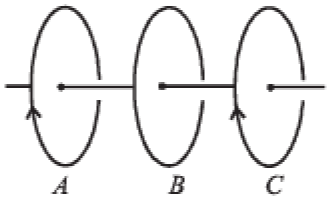
Ans. 1 mV

Solution. KEY CONCEPT : This is based on motional emf.



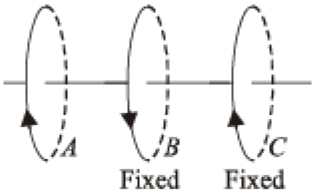
$e = vB\ell = 50 \times 0.2 \times 10^{-4} \times 1 = 10^{-3}$ volt $= 1$ milli volt

Q.3. Three identical closed coils A, B and C are placed with their planes parallel to one another. Coils A and C carry equal currents as shown in Fig. Coils B and C are fixed in position and coil A is moved towards B with uniform motion. Is there any induced current in B ? If no, give reasons. If yes mark the direction of the induced current in the diagram. (1982 - 2 Marks)



Ans. Yes, opposite direction of A

Solution.



NOTE : When the coil A moves towards B, the number of magnetic lines of force passing through B changes.

Therefore, an induced emf and hence induced current is produced in B.

The direction of current in B will be such as to oppose the field change in B and therefore, will be in the opposite direction of A.

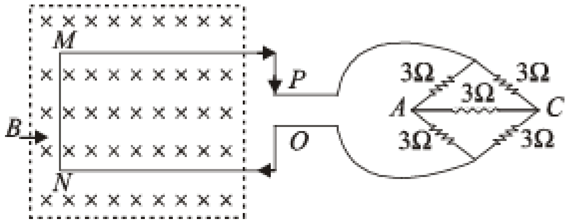
Q.4. A square metal wire loop of side 10 cms and resistance 1ohm is moved with a constant velocity v_0 in a uniform magnetic field of induction B = 2 webers/

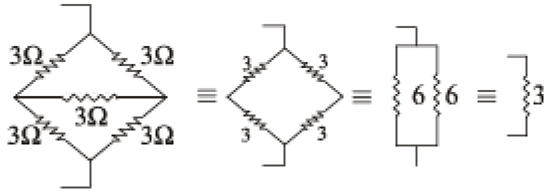
m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 ohms. The resistances of the lead wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 milliampere in the loop ?

Give the direction of current in the loop. **(1983 - 6 Marks)**

Ans. 0.02 m/s, clockwise direction

Solution.





NOTE : The network behaves like a balanced wheatstone bridge.

The free electrons in the portion MN of the rod have a velocity v in the right direction. Applying Fleming's left hand rule, we find that the force on electron will be towards N.

Hence, M will be + ve and N will be negative. Current will flow in clockwise direction.

The induced emf developed is given by

$e = vB\ell = v \times 2 \times 0.1 = 0.2v$ ...(i)

Now, $e = IR$

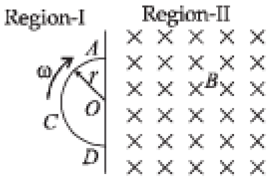
$e = 10^{-3} \times 4 = 4 \times 10^{-3}$ amp ...(ii)

From (i) and (ii),

$0.2\ v = 4 \times 10^{-3}$

$\therefore \quad v = \frac{4 \times 10^{-3}}{0.2} = 0.02\ m/s$

Q.5. Space is divided by the line AD into two regions. Region I is field free and the Region II has a uniform magnetic field B directed into the plane of the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity w about an axis passing through O and the perpendicular to the plane of the paper. The effective resistance of the loop is R. (1985 - 6 Marks)



(i) Obtain an expression for the magnitude of the induced current in the loop.

(ii) Show the direction of the current when the loop is entering into the Region II.

(iii) Plot a graph between the induced e.m.f and the time of rotation for two periods of rotation.

Ans. (i) $\frac{1}{2}\frac{Br^2\omega}{R}$

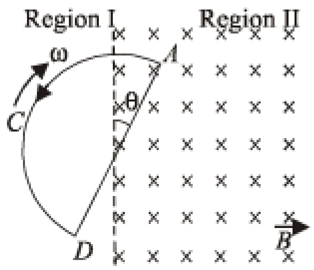
(ii) anticlockwise

Solution. (i) Induced emf

$$E = -\frac{d\phi}{dt} = -\frac{d}{dt}(B \times A)$$

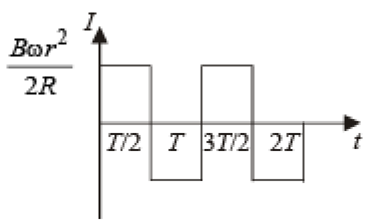
$$= -\frac{d}{dt}\left[B\left(\frac{1}{2}r^2\theta\right)\right] = -\frac{1}{2}Br^2\frac{d\theta}{dt} = -\frac{1}{2}Br^2\omega$$

$\therefore \quad I = \frac{E}{R} = -\frac{1}{2}\frac{Br^2\omega}{R} \Rightarrow |I| = \frac{1}{2}\frac{Br^2\omega}{R}$



(ii) The loop is entering in the magnetic field and hence magnetic lines of force passing through the loop is increasing in the downward direction. Therefore, current will flow in the loop in such a direction which will oppose the change. The current will flow in the anticlockwise direction.

(iii) Graph between induced emf and period of rotation: For first half rotation, (t = T/2), when the loop enters the field, the current is in anticlockwise direction. Magnitude of current remains constant at I = $B\omega r^2/2R$.

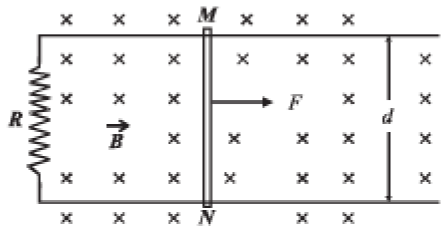


For next half rotation, when the loop comes out of the field, current of the same magnitude is set up clockwise.

Anticlockwise current is supposed to be positive. The I-t graph is shown in the figure for two periods of rotation.

Q.6. Two long parallel horizontal rails, a distance d apart and each having a resistance λ per unit length, are joined at one end by a resistance R. A perfectly conducting rod MN of mass m is free to slide along the rails without friction (see figure). There is a uniform magnetic field of induction B normal to the plane of the paper and directed into the paper.

A variable force F is applied to the rod MN such that, as the rod moves, a constant current flows through R. (1988 - 6 Marks)



(i) Find the velocity of the rod and the applied force F as function of the distance x of the rod from R.

(ii) What fraction of the work done per second by F is converted into heat ?

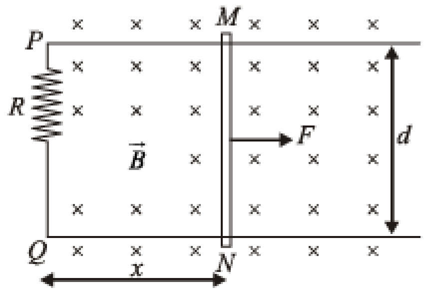
Ans. (i) $V = \left(\frac{R+2\lambda x}{Bd}\right)I,\ F = BId + \frac{2\lambda m I^2}{(Bd)^2}(R+2\lambda x)$,

(ii) $\left[1+\frac{2\lambda m I(R+2\lambda x)}{B^3 d^3}\right]^{-1}$

Solution. (i) A variable force F is applied to the rod MN such that as the rod moves in the uniform magnetic field a constant current flows through R. Consider the loop MPQN.

Let MN be at a distance x from PQ.

Length of rails in loop = 2x



∴ Resistance of rails in loop = 2xλ

∴ Total resistance of loop = R + 2λx

Induced emf = Bvd

$\therefore$ Induced current $(I) = \frac{Bvd}{R+2\lambda x}$

So for constant I,

$$v = \frac{(R+2\lambda x)}{Bd} I \quad ..(i)$$

Furthermore, as due to induced current I the wire will experience a force F_M = BId opposite to its motion, the equation of motion of the wire will be

$F - F_M = ma$ i.e., $F = F_M + ma$

But as here F_M = BId and from equation (i)

$$a = \frac{dv}{dt} = \frac{2\lambda I}{Bd}\frac{dx}{dt} = \frac{2\lambda Iv}{Bd} = \frac{2\lambda I^2}{(Bd)^2}(R+2\lambda x)$$

$$\text{So, } F = BId + \frac{2\lambda m I^2}{(Bd)^2}(R+2\lambda x)$$

(ii) As the work done by force F per sec.

$$\frac{dW}{dt} = P = Fv = \left[BId + \frac{2\lambda m I^2}{(Bd)^2}(R+2\lambda x)\right]\left[\frac{R+2\lambda x}{Bd}.I\right]$$

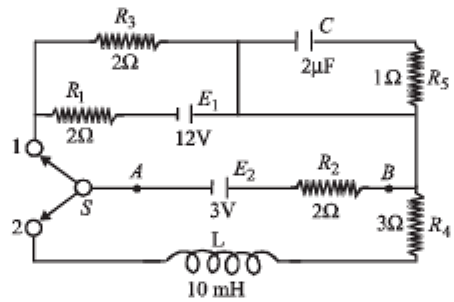
$$\text{i.e., } P = \left[I^2(R+2\lambda x) + \frac{2\lambda m I^3}{B^3 d^3}(R+2\lambda x)^2\right]$$

and heat produced per second, i.e., joule heat

$$H = I^2(R+2\lambda x)$$

$$\text{So, } f = \frac{H}{P} = \left[1 + \frac{2\lambda m I(R+2\lambda x)}{B^3 d^3}\right]^{-1}$$

Q.7. A circuit containing a two position switch S is shown in fig. (1991 - 4 + 4 Marks)



(a) The switch S is in position '1'. Find the poten tial difference $V_A - V_B$ and the rate of production of joule heat in R_1.

(b) If now the switch S is put in position 2 at t = 0 find

(i) steady current in R_4 and

(ii) the time when current in R_4 is half the steady value.

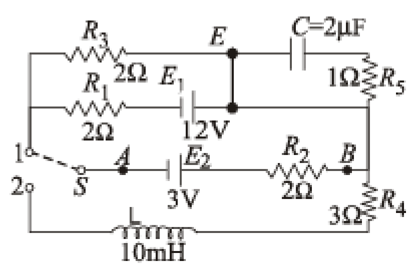
Also calculate the energy stored in the inductor L at that time

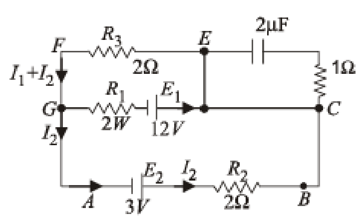
Ans. (a) –5V, 24.5 W

(b) (i) 0.6 amp.

(ii) 1.386×10^{-3} sec., 4.5×10^{-4} J

Solution. (a) (i) In this case S and I are connected.





Using Kirchhoff's law in ABCDGA

$+ 3 - I_2 \times 2 - 12 + I_1 \times 2 = 0$

$2 I_1 - 2I_2 = 9$ ...(i)

Applying Kirchhoff 's law in DEFGD

$- 2I_1 + 12 - (I_1 + I_2) 2 = 0$

$\Rightarrow 2I1 + I2 = 6$ ...(ii)

From (i) and (ii) $I_1 = \frac{21}{6}$ amp.

∴ From (ii) $I_2 = -1$ amp.

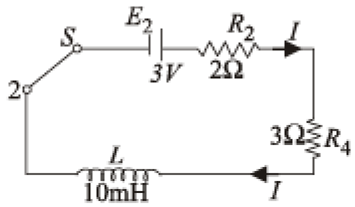
To find potential difference between A and B

$V_A + 3 - (-1) \times 2 = V_B \Rightarrow V_A - V_B = -5V$

The rate of production of heat in R_1

$= I_1^2 R_1 = \left(\frac{21}{6}\right)^2 \times 2 = 24.5W$

(b) (i) When the switch is put in position 2 then the active circuit will be as shown in the figure.



When the steady state current is reached then the inductor plays no role in the circuit

$E_2 = I (R_2 + R_4)$

$\Rightarrow \quad I = \frac{3}{5} = 0.6 \text{ amp.}$

(ii) KEY CONCEPT : The growth of current in L–R circuit is given by the expression

$$I = I_0\left[1 - e^{-\frac{R}{L}t}\right]$$

$$\text{When } I = \frac{I_0}{2}, \text{ then } \frac{I_0}{2} = I_0\left[1 - e^{-\frac{R}{L}t}\right]$$

$$\Rightarrow \frac{1}{2} = 1 - e^{\frac{-R}{L}t} \quad \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides

$$\log_e e^{-\frac{R}{L}t} = \log_e \frac{1}{2}$$

$$\Rightarrow \frac{R}{L}t = 0.693 \Rightarrow t = 0.693\frac{L}{R} = \frac{0.6930 \times 10 \times 10^{-3}}{(2+3)}$$

when R = $R_2 + R_4$

$\Rightarrow$ t = 1.386 × 10^{-3} sec.

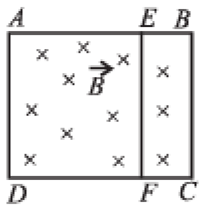
Thus this much time is required for current to reach half of its steady value.

The energy stored by the inductor at that time is given

$$\text{by } E = \frac{1}{2}LI^2 = \frac{1}{2} \times 10 \times 10^{-3} \times \left(\frac{0.6}{2}\right)^2 = 4.5 \times 10^{-4}\text{J}$$

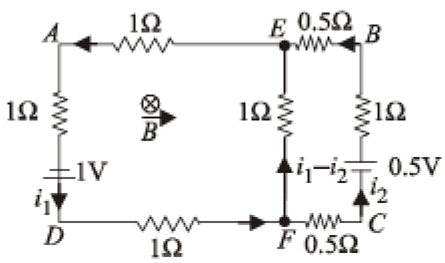
Q.8. A rectangular frame ABCD, made of a uniform metal wire, has a straight connection between E and F made of the same wire, as shown in Fig. AEFD is a square of side 1m, and EB = FC = 0.5m. The entire circuit is placed in steadily increasing, uniform magnetic field directed into the plane of the paper and normal to it.

The rate of change of the magnetic field is 1T/s. The resistance per unit length of the wire is 1Ω / m. Find the magnitudes and directions of the currents in the segments AE, BE and EF. (1993-5 Marks)



Ans. $\frac{7}{22}$ amp, $\frac{6}{22}$ amp, $\frac{1}{22}$ amp

Solution. The equivalent circuit is drawn in the adjacent figure.



NOTE : As the magnetic field increases in the downward direction, an induced emf will be produced in the AEFD as well as in the circuit EBCF such that the current flowing in the loop creates magnetic lines of force in the upward direction (to the plane of paper).

Thus, the current should flow in the anticlockwise direction in both the loops.

Induced emf in loop AEFD

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA = -A\frac{dB}{dt} = -1 \times 1 = -1\text{volt}$$

Induced emf in loop EBCF

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}BA' = -A'\frac{dB}{dt} = -0.5 \times 1 = -0.5 \text{ volt}$$

Let the current flowing in the branch EADF be i_1 and the current flowing in the branch FCBE be i_2. Applying junction law at F, we get current in branch FE to be

$(i_1 - i_2)$ Applying Kirchhoff 's law in loop EADFE

$-1 \times i_1 - 1 \times i_1 + 1 - 1 \times i_1 - 1(i_1 - i_2) = 0$

$\Rightarrow 4i_1 - i_2 = 1$ ... (i)

Applying Kirchhoff 's law in loop EBCFE

$+0.5i_1 - 0.5 + 1i_2 + 0.5i_2 - 1(i_1 - i_2) = 0$

$-i_1 + 3i_2 = 0.5$ ... (ii)

Solving (i) and (ii)

$11i_1 = 3.5$

$\Rightarrow \quad i_1 = 3.5/11 = \frac{7}{22}A$

Also $11i_2 = 3$

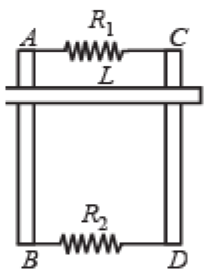
$\Rightarrow \quad i_2 = 3/11\,A = \frac{6}{22}A$

$\therefore$ Current in segment $AE = i_1 = \frac{7}{22}A$

Current in segment $BE = i_2 = \frac{6}{22}A$

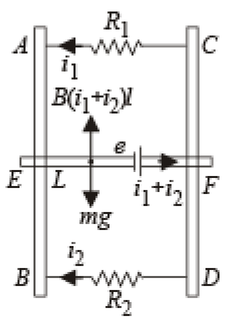
Current in segment $EF = (i_1 - i_2) = \frac{1}{22}A$

Q.9. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at two ends by resistances R_1 and R_2 as shown in Figure. A horizontal metallic bar L of mass 0.2 kg slides without friction vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6 Tesla perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the powers dissipated in R_1 and R_2 are 0.76 Watt and 1.2 watt respectively. Find the terminal velocity of the bar L and the values of R_1 and R_2. (1994 - 6 Marks)



Ans. 1 m/s, 0.47 Ω, 0.3 Ω

Solution. KEY CONCEPT : We can understand the direction of flow of induced currents by imagining a fictitious battery to be attached between E and F. The direction of induced current can be found with the help of Lenz's law.



NOTE : P. d. across parallel combinations remains the same

Also, $P_1 = ei_1 = 0.76$ W and $P_2 = ei_2 = 1.2$ W

$$\therefore \quad \frac{i_1}{i_2} = \frac{1.76}{1.2} \Rightarrow i_1 = \frac{1.76}{1.2} i_2 \qquad \ldots \text{(ii)}$$

The horizontal metallic bar L moves with a terminal velocity.

This means that the net force on the bar is zero.

$\therefore$ B $(i_1 + i_2)$ = mg

$$\Rightarrow \quad i_1 + i_2 = \frac{mg}{B\ell} = \frac{0.2 \times 9.8}{0.6 \times 1} = \frac{49}{15} \text{ amp.} \quad \text{......(iii)}$$

From (ii) and (iii)

$$\frac{1.76}{1.2}i_2 + i_2 = \frac{49}{15}$$

$$\Rightarrow i_2 = 2 \text{ amp.} \Rightarrow i_1 = \frac{19}{15} \text{amp.} \Rightarrow e = \frac{0.76}{19/15} = 0.6V$$

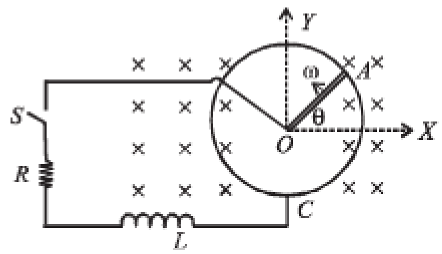
The induced emf across L due to the movement of bar L in a magnetic field

$$e = Bv_T L \Rightarrow v_T = \frac{e}{BL} = \frac{0.6}{0.6 \times 1} = 1\text{m/s}$$

Also from (i),

$$R_1 = \frac{e}{i_1} = \frac{0.6}{19/15} = 0.47\Omega \text{ and } R_2 = \frac{e}{i_2} = \frac{0.6}{2} = 0.3\Omega$$

Q.10. A metal rod OA of mass 'm' and length 'r' is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction $\vec{B}$ is applied perpendicular and into the plane of rotation as shown in the figure below. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open. (1995 - 10 Marks)



(a) What is the induced emf across the terminals of the switch?

(b) The switch S is closed at time t = 0.

(i) Obtain an expression for the current as a function of time.

(ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at t = 0.

Ans. (a) $\frac{Br^2\omega}{2}$ (b) $I=\frac{B\omega r^2}{2R}\left[1-e^{-\left(\frac{R}{L}\right)t}\right]$, $\tau=\frac{B^2r^4w}{4R}+\frac{mgr}{2}\cos\omega t$

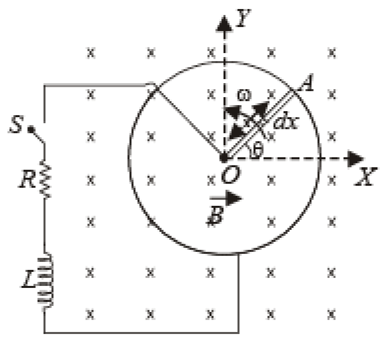
Solution. (i) (a) Let us consider a small length of metal rod dx at a distance x from the origin. Small amount of emf (de) induced in this small length (due to metallic rod cutting magnetic lines of force) is

de = B (dx) v … (i)

where v is the velocity of small length

dxv = xω … (ii)

∴ The total emf acoss the whole metallic rod OA is



$$e=\int_0^r Bx\omega\, dx = B\omega\left[\frac{x^2}{2}\right]_0^r=\frac{Br^2\omega}{2}$$

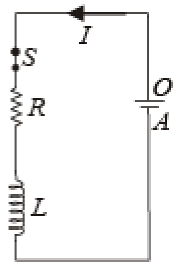
(b) The above diagram can be reconstructed as the adjacent figure. e is a constant. O will accumulate positive charge and A negative. When the switch S is closed, transient current at any time t, when current I is flowing in the circuit,

Here, $I=I_0(1-e^{t/\tau})$

$$I_0=\frac{e}{R}=\frac{B\omega r^2}{2R}$$

and $\tau=\frac{L}{R}$

Therefore, $I = \frac{B\omega r^2}{2R}[1 - e^{-\left(\frac{R}{L}\right)t}]$



(ii) In steady state,

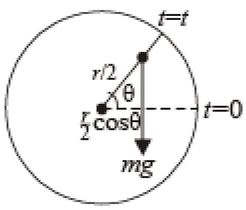
$I = \frac{B\omega r^2}{2R}$ [$\because$ t has a large value and $e^{-\left(\frac{R}{L}\right)t} \to 0$]

NOTE : When current flows in the circuit in steady state, there is a power loss through the resistor.

Also since the rod is rotating in a vertical plane, work needs to be done to keep it at constant angular speed.

Power loss due to current I will be

$P = I^2 R = \left(\frac{Br^2\omega}{2R}\right)^2 R$



If torque required for this power is τ_1 then

$P = \tau_1\omega$

$\Rightarrow \quad \tau_1 = \frac{B^2 r^4 \omega}{4R}$

Torque required to move the rod in circular motion against gravitational field

$$\tau_2 = mg \times \frac{r}{2} \cos\theta$$

The total torque

$\tau = \tau_1 + \tau_2$ (Clockwise)

$$\tau = \frac{B^2 r^4 \omega}{4R} + \frac{mgr}{2} \cos \omega t$$

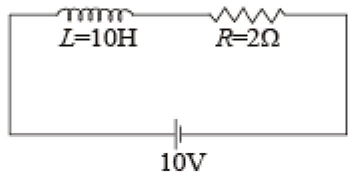
The required torque will be of same magnitude and in anticlockwise direction. The second term will change signs as the value of cos θ can be positive as well as negative.

Subjective Questions Part -2

Q.11. A solenoid has an inductance of 10 henry and a resistance of 2 ohm. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach 1/4 of its maximum value? (1996 - 3 Marks)

Ans. 3.466 sec

Solution. KEY CONCEPT : Let I_0 be the current at steady state. The magnetic energy stored in the inductor at this state will be



$$E = \frac{1}{2}LI_0^2 \qquad \ldots(i)$$

This is the maximum energy stored in the inductor. The current in the circuit for one fourth of this energy can be found as

$$\frac{1}{4} \times E = \frac{1}{2}LI^2 \qquad \ldots(ii)$$

Dividing equation (i) and (ii)

$$\frac{E}{E/4} = \frac{\frac{1}{2}LI_0^2}{\frac{1}{2}LI^2} \Rightarrow I = \frac{I_0}{2}$$

Also, $V = I_0 R$

$$\Rightarrow \quad I_0 = \frac{V}{R} = \frac{10}{2} = 5 \text{ amp.} \qquad \therefore\ I = \frac{I_0}{2} = \frac{5}{2} = 2.5 \text{ amp.}$$

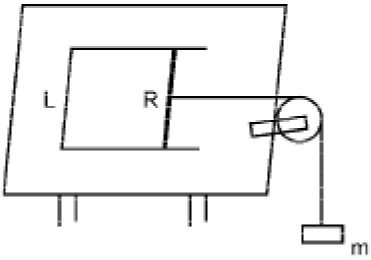
The equation for growth of current in L-R circuit is

$I = I_0 [1 - e^{-Rt/L}]$

$$\Rightarrow \quad 2.5 = 5\,[1 - e^{-2t/10}] \quad \Rightarrow \quad \frac{1}{2} = 1 - e^{-t/5}$$

$\Rightarrow$ t = 5 $\log_e 2$ = 2 × 2.303 × 0.3010 = 3.466 sec.

Q.12. A pair of parallel horizontal conducting rails of negligible resistance shorted at one end is fixed on a table. The distance between the rails is L. A conducting massless rod of resistance R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate. (1997 - 5 Marks)



Ans. (i) $\frac{mgR}{B^2L^2}$ (ii) $\frac{g}{2}$

Solution. KEY CONCEPT : If v is the velocity of the rod at any time t, induced emf is BvL and so induced current in the rod

$$I = \frac{\text{Induced e.m.f.}}{R} = \frac{BvL}{R}$$

Due to this current, the rod in the field B will experience a force

$$F = BIL = \frac{B^2L^2v}{R} \text{ (opposite to its motion) } \ldots (1)$$

So, equation of motion of the rod will be,

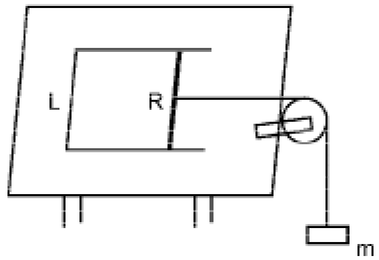
T – F = 0 × a, i.e., T = F [as rod is massless]

$$mg - T = ma \Rightarrow a = g - \frac{T}{m} = g - \frac{B^2L^2v}{mR} \quad \ldots (2)$$

So rod will acquire terminal velocity when its acceleration is zero i.e.,

$$g - \frac{B^2L^2v_T}{mR} = 0 \text{ i.e. } v_T = \frac{mgR}{B^2L^2};$$

For the case when velocity is $\frac{v_T}{2}$

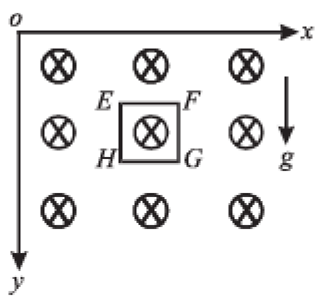


$$v = \frac{v_T}{2} = \frac{mgR}{2B^2L^2}$$

Substituting this value of velocity in eq. (2) we get

$$a = g - \frac{B^2L^2}{mR} \times \frac{1}{2}\frac{mgR}{B^2L^2} = g - \frac{1}{2}g = \frac{g}{2}$$

Q.13. A magnetic field $B = B_0\,(y/a)\,\hat{k}$ is into the paper in the +z direction. B_0 and a are positive constants. A square loop EFGH of side a, mass m and resistance R, in x – y plane, starts falling under the influence of gravity see figure) Note the directions of x and y axes in figure. (1999 - 10 Marks)



Find

(a) the induced current in the loop and indicate its direction.
(b) the total Lorentz force acting on the loop and indicate its direction, and

(c) an expression for the speed of the loop, v(t) and its terminal value.

Ans. (a) $\frac{B_0 a v(t)}{R}$, anticlockwise (b) $-\frac{B_0^2 a^2 v(t)}{R}$, upward

Solution. Suppose at t = 0, y = 0 and t = t, y = y (a)

Total magnetic flux $= \vec{B}.\vec{A}$

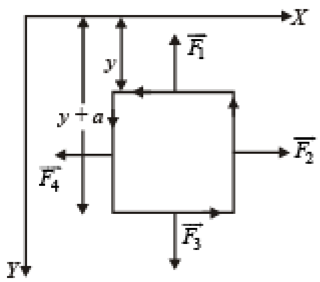
where $\vec{A} = a^2 \hat{k}$ and $\vec{B} = \frac{B_0 y}{a}\hat{k}$

$\therefore \quad \phi = \frac{B_0 y}{a}.a^2 = B_0 ya$

Net emf, $e = -\frac{d\phi}{dt} = -B_0 a\frac{dy}{dt} = -B_0 av(t)$

As total resistance = R

$\therefore |i| = \frac{|e|}{R} = \frac{B_0 av(t)}{R}$



NOTE : Now as loop goes down, magnetic flux linked with it increases, hence induced current flows in such a direction so as to reduce the magnetic flux linked with it. Hence, induced current flows in anticlockwise direction.

(b) Each side of the cube will experience a force as shown (since a current carrying segment in a magnetic field experiences a force).

$$\vec{F}_1 = i(\vec{\ell}\times\vec{B}) = i\left(-a\hat{i}\times\frac{B_0 y}{a}\hat{k}\right) = B_0 y(\hat{i}\times\hat{j});$$

$$\vec{F}_3 = i\left(+a\hat{i}\times\frac{B_0(y+a)}{a}\hat{k}\right) = iB_0(y+a)\,\hat{j}$$

NOTE : $\vec{F}_2 = -\vec{F}_4$ and hence will cancel out each other..

Net force, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = -iB_0\, a\hat{j} = -\frac{B_0^2 a^2 v(t)}{R}\hat{j}$

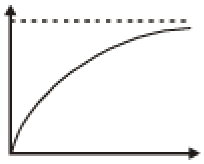
(c) Total net force $= mg\,\hat{j} + \vec{F} = \left[mg - \frac{B_0^2 a^2 v(t)}{R}\right]\hat{j};$

$$\therefore \quad m\frac{dv}{dt} = mg - \frac{B_0^2 a^2 v(t)}{R}$$

Integrating it, we get, $\int_0^v \frac{dv}{g - \frac{B_0^2 a^2 v(t)}{mR}} = \int_0^t dt$

$$\frac{\log\left[g - \frac{B_0^2 a^2 v(t)}{mR}\right]_0^{(v)t}}{\frac{-B_0^2 a^2}{mR}} = t$$

or $\quad \log\left[\frac{g - \frac{B_0^2 a^2 v(t)}{mR}}{g}\right] = -\frac{B_0^2 a^2 t}{mR}$



or $\quad 1 - \frac{B_0^2 a^2 v(t)}{mgR} = e^{-\left(B_0^2 a^2 t\right)/mR}$

or $$1-\frac{B_0^2 a^2 v(t)}{mgR}=e^{-\left(B_0^2 a^2 t\right)/mR}$$

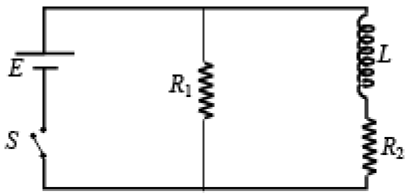
or $$1-e^{-\left(B_0^2 a^2 t\right)/mR}=\frac{B_0^2 a^2}{mgR}v(t);$$

$$\therefore \quad v(t)=\frac{mgR}{B_0^2 a^2}\left[1-e^{-\left(B_0^2 a^2 t\right)/mR}\right]$$

When terminal velocity is attained, v (t) does not depend on t

$$\therefore \quad v(t)=\frac{mgR}{B_0^2 a^2}$$

Q.14. An inductor of inductance L = 400 mH and resistors of resistances R_1 = 2Ω and R_2 = 2 Ω are connected to a battery of emf E = 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at time t = 0. What is the potential drop across L as a function of time? After the steady state is reached, the switch is opened. What is the direction and the magnitude of current through R1 as a function of time? (2001-5 Marks)



Ans. $12e^{-5t}$V, $3e^{-10t}$A, clockwise

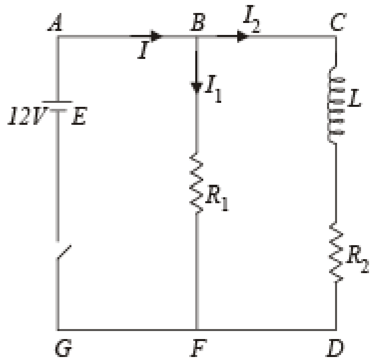
Solution. This is a question on growth and rise of current.

GROWTH OF CURRENT : Let at any instant of time t the current be as shown in the figure.

Applying Kirchoff's law in the loop ABCDFGA we get, starting from G moving clockwise

$$E-L\frac{dI_2}{dt}-I_2R_2=0$$

$$\text{or } I_2 = \frac{E}{R_2}\left[1 - e^{-\frac{R}{L}t}\right]$$



Also we know that the emf (V) produced across the inductor

$$V = -\frac{d\phi}{dt} = -\frac{d}{dt}[LI_2] = -L\frac{dI_2}{dt}$$

$$= -L\frac{d}{dt}\left[\frac{E}{R_2}\left(1 - e^{\frac{-R_2}{L}t}\right)\right]$$

$V = -E\, e^{-\frac{R_2}{L}t}$. Here the negative sign shows the opposition to the growth of current.

$$\therefore \quad V = 12 e^{-\frac{2}{400\times10^{-3}}t} = 12\, e^{-5t} \text{ volt}$$

DECAY OF CURRENT : When the switch is opened, the branch AG is out of the circuit. Therefore, the current decays through the circuit CBFDC (in clockwise direction).

Applying Kirchhoff 's law

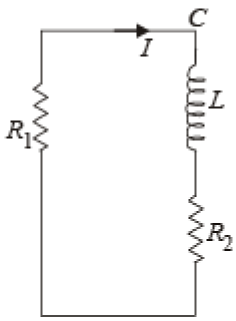
$$I(R_1+R_2)-\left(-\frac{L\,dI}{dt}\right)=0$$

$$\therefore \quad \frac{dI}{I}=-\left(\frac{R_1+R_2}{L}\right)dt$$

$\therefore$ On integrating,

$$\int_{I_0}^{I}\frac{dI}{I}=-\frac{(R_1+R_2)}{L}\int_0^t dt$$

$$\therefore \quad I=I_0 e^{-\frac{(R_1+R_2)t}{L}}$$



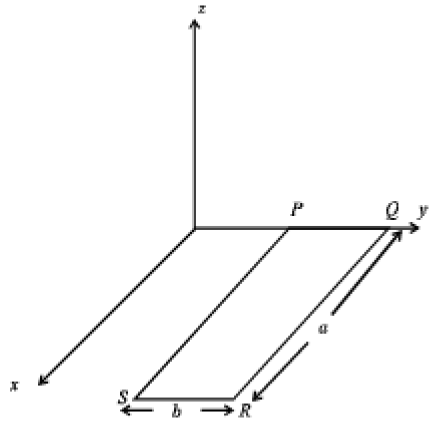
Here, $\frac{R_1+R_2}{L}=\frac{2+2}{400\times 10^{-3}}=10$

and $I_0=\frac{E}{R_1+R_2}=\frac{12}{4}=3\text{ A}$

$\therefore \quad I=3e^{-10t}A$, clockwise.

Alternatively, you may directly find the time constant $\tau=\frac{L}{R_1+R_2}$ and use the equation $i = i_0 e^{-t/\tau}$ where $i_0 = 6A$

Q.15. A rectan gular loop PQRS made fr om a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B}=(3\hat{i}+4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. (2002 - 5 Marks)



(a) What is the direction of the current I in PQ?

(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.

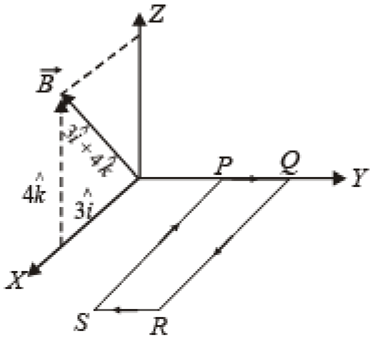
Ans. (a) P to Q

(b) $IbB_0(3\hat{k}-4\hat{i})$

(c) $I=\dfrac{mg}{6aB_0}$

Solution. Let us consider the current in the clockwise direction in loop PQRS. Force on wire QR,

$$\vec{F}_{QR}=I(\vec{\ell}\times\vec{B}) = I[(a\hat{i})\times(3\hat{i}+4\hat{k})B_0]$$
$$= IB_0[3a\hat{i}\times\hat{i}+4a\hat{i}\times\hat{k}] = IB_0[0+4a(-\hat{j})] = -4aB_0I\hat{j}$$



Force on wire PS

$$\vec{F}_{PS} = I(\vec{\ell} \times \vec{B}) = I[a(-\hat{i}) \times (3\hat{i} + 4\hat{k}) B_0] = 4aB_0 I \hat{j}$$

Thus we see that force on QR is equal and opposite to that on PS and balance each other.

The force on RS is

$$\vec{F}_{RS} = I(\vec{\ell} \times \vec{B}) = I[b(-\hat{j}) \times (3\hat{i} + 4\hat{k}) B_0]$$

$$= I b B_0 [3\hat{k} - 4\hat{i}] \quad \text{... (i)}$$

The torque about PQ by this force is

$$\vec{\tau}_{RS} = \vec{r} \times \vec{F} = (\hat{i}\, a) \times (3\hat{k} - 4\hat{i})\, IbB_0$$

$$= I\, abB_0 (3\hat{j}) \quad \text{.. (ii)}$$

The torque about PQ due to weight of the wire PQRS is

$$\tau = mg\left(\frac{a}{2}\right) \quad \text{... (iii)}$$

For the wire loop to be horizontal, we have to equate (ii) and

(iii) $3IabB_0 = mg\frac{a}{2}$

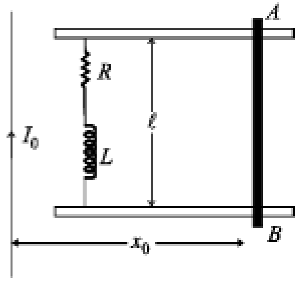
$$\Rightarrow \quad I = \frac{mg}{6bB_0} \quad \text{... (iv)}$$

Therefore, (a) Th e direction of current assumed is right . This is because torque due to mg and current are in opposite directions. Therefore, current is from P to Q.

(b) From (i), $\vec{F}_{RS} = IbB_0(3\hat{k} - 4\hat{i})$

(c) From (iv), $I = \dfrac{mg}{6aB_0}$

Q.16. A metal bar AB can slide on two parallel thick metallic rails separated by a distance ℓ. A resistance R and an inductance L are connected to the rails as shown in the figure. A long straight wire carrying a constant current I_0 is placed in the plane of the rails and perpendicular to them as shown. The bar AB is held at rest at a distance x_0 from the long wire. At t = 0, it is made to slide on the rails away from the wire. Answer the followin g questions. (2002 - 5 Marks)



(a) Fin d a relation amon $i, \frac{di}{dt}$ and $\frac{d\phi}{dt}$, **where i is the current in the circuit and f is the flux of the magnetic field due to the long wire through the circuit.**

(b) It is observed that at time t = T, the metal bar AB is at a distance of $2x_0$ from the long wire and the resistance R carries a current i1. Obtain an expression for the net charge that has flown through resistance R from t = 0 to t = T.

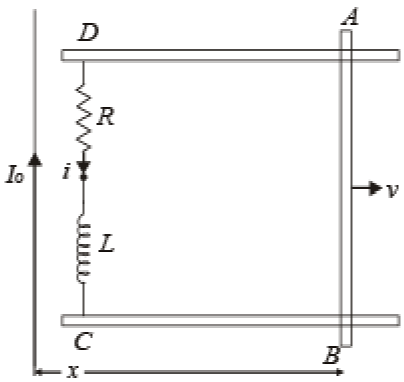
(c) The bar is suddenly stopped at time T. The current through resistance R is found to be $\frac{i_1}{4}$ at time $2T$. **Find the value of L/R in terms of the other given quantities.**

Ans. (a) $\frac{d\phi}{dt} = iR + L\frac{di}{dt}$

(b) $\frac{1}{R}\left[\frac{\mu_0 I_0 \ell}{2\pi}\log_e 2\right] - \frac{L}{R} i_1$

(c) $\frac{T}{2\log_e 2}$

Solution. (a) KEY CONCEPT : As the metal bar AB moves towards the right, the magnetic flux in the loop ABCD increases in the downward direction. By Lenz's law, to oppose this, current will flow in anticlockwise direction as shown in figure.



Applying Kirchhoff 's loop law in ABCD, we get

$$\frac{d\phi}{dt} = iR + L\frac{di}{dt} \quad \text{...(i)}$$

(b) Let AB be at a distance x from the long straight wire at any instant of time t during its motion. The magnetic field at that instant at AB due to long straight current carrying wire is

$$B = \frac{\mu_0 I_0}{2\pi x}$$

The change in flux through ABCD in time dt is

dφ = B (dA) = Bℓdx

Therefore, the total flux change when metal bar moves from a distance x_0 to $2x_0$ is

$$\Delta\phi = \int_{x_0}^{2x_0} B\ell dx = \ell\int_{x_0}^{2x_0} \frac{\mu_0 I_0}{2\pi x} dx = \frac{\mu_0 I_0 \ell}{2\pi}[\log_e x]_{x_0}^{2x_0}$$

$$= \frac{\mu_0 I_0 \ell}{2\pi}\log_e 2 \quad \text{... (ii)}$$

The charge flowing through resistance R in time T is

$$q=\int_0^T i dt=\int_0^T \frac{1}{R}\left[E_{\text{induced}}-L\frac{di}{dt}\right]dt \quad \text{[from eq. (i)]}$$

$$=\frac{1}{R}\int_0^T E_{\text{induced}}dt-\frac{L}{R}\int_0^{i_1} d\,i=\frac{1}{R}(\Delta\phi)-\frac{L}{R}i_1$$

$$q=\frac{1}{R}\left[\frac{\mu_0 I_0 \ell}{2\pi}\log_e 2\right]-\frac{L}{R}i_1 \qquad \text{from eq. (ii)}$$

(c) When the metal bar AB is stopped, the rate of change of magnetic flux through ABCD becomes zero.

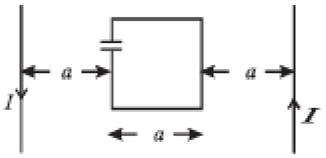
From (i),

$$iR=-L\frac{di}{dt}$$

$$\int_T^{2T} dt=\frac{L}{R}\int_{i_1}^{i_1/4}\frac{di}{i}$$

$$T=-\frac{L}{R}\log_e\frac{i_1/4}{i_1} \Rightarrow \frac{L}{R}=\frac{T}{2\log_e 2}$$

Q.17. A square loop of side 'a' with a capacitor of capacitance C is located between two current carrying long parallel wires as shown. The value of I in the wires is given as I = I_0sinωt. (2003 - 4 Marks)



(a) Calculate maximum current in the square loop.

(b) Draw a graph between charges on the upper plate of the capacitor vs time.

Ans. (a) $\frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$

Solution. (a) Let us consider a small strip of thickness dx as shown in the figure.

The magnetic field at this strip

$B = B_A + B_B$

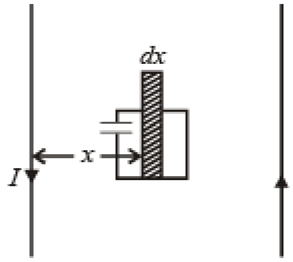
(Perpendicular to the plane of paper directed upwards)

$$= \frac{\mu_0}{2\pi}\frac{I}{x} + \frac{\mu_0}{2\pi}\frac{I}{(3a-x)}$$

B_A = Magnetic field due to current in wire A

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} + \frac{1}{3a-x}\right]$$

B_B = Magnetic field due to current in wire B



Small amount of magnetic flux passing through the strip of thickness dx is

$$d\phi = B \times a dx = \frac{\mu_0 I a \times 3a\, dx}{2\pi x(3a-x)}$$

Total flux through the square loop

$$\phi = \int_a^{2a} \frac{\mu_0 I \times 3a^2}{2\pi} \frac{dx}{x(3a-x)} = \frac{\mu_0 I a}{\pi} \ln 2$$

$$= \frac{\mu_0 a \ln(2)}{\pi} (I_0 \sin \omega t)$$

The emf produced

$$e = \left| -\frac{d\phi}{dt} \right| = \frac{\mu_0 a I_0 \omega}{\pi} \ln(2) \cos \omega t$$

Charge stored in the capacitor

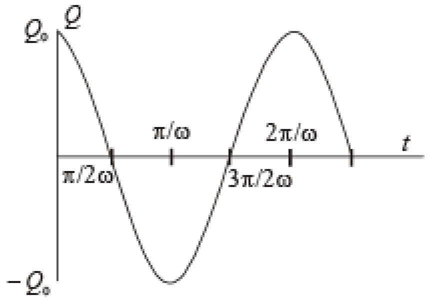
$$q = C \times e = C \times \frac{\mu_0 a I_0 \omega}{\pi} \ln(2) \cos \omega t \quad \text{... (i)}$$

∴ Current in the loop

$$i = \frac{dq}{dt} = \frac{C \times \mu_0 a I_0 \omega^2}{\pi} \ln(2) \sin \omega t$$

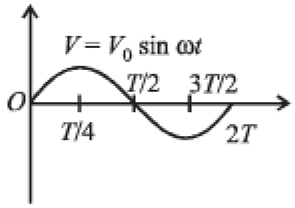
$$\therefore \quad i_{max} = \frac{\mu_0 a I_0 \omega^2 C \ln(2)}{\pi}$$

(b) From (i), the graph between charge and time is



$$\text{Here, } q_0 = \frac{C \times \mu_0 a I_0 \omega \ln(2)}{\pi}$$

Q.18. In a series L–R circuit (L = 35 mH and R = 11 Ω), a variable emf source ($V = V_0 \sin \omega t$) of V_{rms} = 220 V and frequency 50 Hz is applied. Find the current amplitude in the circuit and phase of current with respect to voltage. Draw current-time graph on given graph (π = 22/7). (2004 - 4 Marks)



Ans. $20\text{A}, \frac{\pi}{4}$

Solution.

Given, $V_{rms} = 220\,V$

$\nu = 50\,Hz, L = 35\,mH, R = 11\Omega$

Impedance

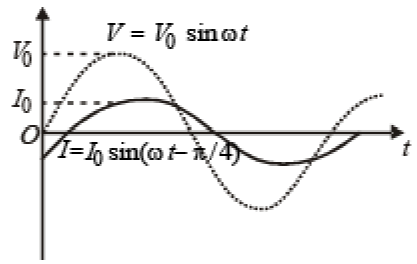
$$Z = \sqrt{(\omega L)^2 + R^2} = 11\sqrt{2}\,\Omega$$

also, $I_0 = \dfrac{V_0}{Z}$

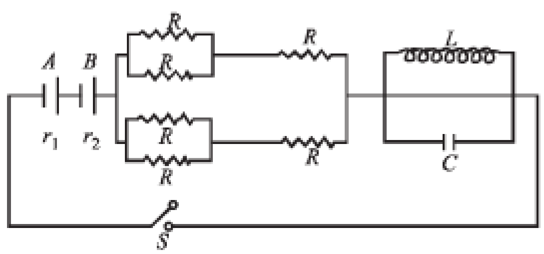
$V_0 = V_{rms}\sqrt{2}$ $\quad\quad \therefore\ I_0 = \dfrac{V_{rms}\sqrt{2}}{Z} = 20A$

$\cos\phi = \dfrac{R}{Z} = \dfrac{1}{\sqrt{2}}$ $\quad\quad \therefore\ \phi = \dfrac{\pi}{4}$

∴ graph is given by.



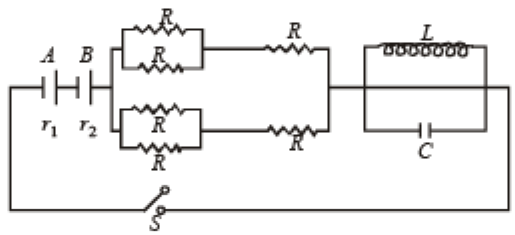
Q.19. In the figure both cells A and B are of equal emf. Find R for which potential difference across battery A will be zero, long time after the switch is closed. Internal resistance of batteries A and B are r1 and r2 respectively (r_1 > r_2). (2004 - 4 Marks)



Ans. $\frac{4}{3}(r_1 - r_2)$

Solution. NOTE : After a long time capacitor will be fully charged, hence no current will flow through capacitor and all the current will flow from inductor. Since current is D.C., resistance of L is zero.

$$\therefore \quad R_{eq} = \left(\frac{R}{2} + R\right) \times \frac{1}{2} + r_1 + r_2 = \frac{3R}{4} + r_1 + r_2$$



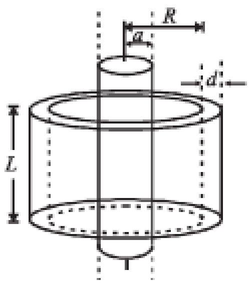
$$I = \frac{\varepsilon + \varepsilon}{R_{eq}} \Rightarrow I = \frac{2\varepsilon}{R_{eq}} = \frac{2\varepsilon}{(3R/4) + r_1 + r_2}$$

Potential drop across A is

$$\varepsilon - Ir_1 = 0 \quad \Rightarrow \quad \varepsilon = \frac{2\varepsilon}{(3R/4) + r_1 + r_2} r_1$$

$$\Rightarrow \quad r_1 = r_2 + 3R/4 \quad \text{or} \quad R = \frac{4}{3}(r_1 - r_2)$$

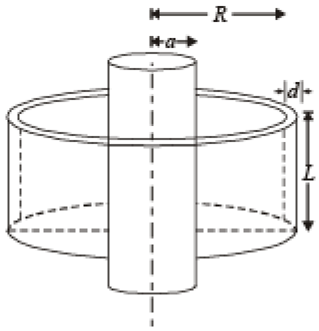
Q.20. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R. thickness d (d << R) and length L. A variable current i = i0 sinwt flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell. (2005 - 4 Marks)



Ans. $I = \frac{\mu_0 n a^2 L d i_0 \omega \cos \omega t}{2\rho R}$

Solution. KEY CONCEPT : The magnetic field in the solenoid is given by

$B = \mu_0 ni$



$\Rightarrow B = \mu_0 ni_0 \sin \omega t$ [$\because i = i_0 = \sin \omega t$ (given)]

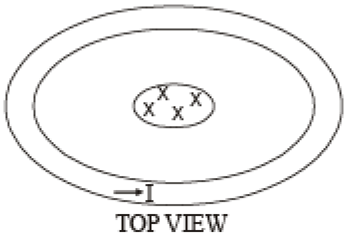
The magnetic flux linked with the solenoid

$\phi = \vec{B}.\vec{A} = BA \cos 90° = (\mu_0 ni_0 \sin \omega t)(\pi a^2)$

$\therefore$ The rate of change of magnetic flux through the solenoid

$$\frac{d\phi}{dt} = \pi \mu_0 n a^2 i_0 \omega \cos \omega t$$

The same rate of change of flux is linked with the cylindrical shell. By the principle of electromagnetic induction, the induced emf produced in the cylindrical shell is



TOP VIEW

$$\varepsilon = -\frac{d\phi}{dt} = -\pi \mu_0 n a^2 i_0 \omega \cos \omega t \quad \text{...(i)}$$

The resistance offered by the cylindrical shell to the flow of induced current I will be

$$R = \rho \frac{\ell}{A}$$

Here, $\ell = 2\pi R$ and $A = L \times d$

$$\therefore \quad R = \rho \frac{2\pi R}{Ld} \qquad \text{... (ii)}$$

The induced current I will be

$$I = \frac{|e|}{R} = \frac{[\pi \mu_0 n a^2 i_0 \omega \cos \omega t] \times Ld}{\rho \times 2\pi R}$$

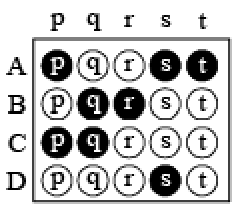
$$\Rightarrow \quad I = \frac{\mu_0 n a^2 Ld\, i_0 \omega \cos \omega t}{2\rho R}$$

Match the Following

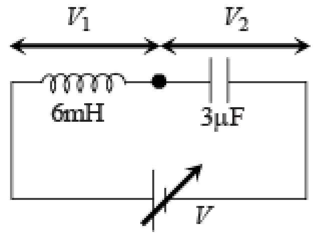
DIRECTIONS : Each question contains statements given in two columns, which have to be matched.

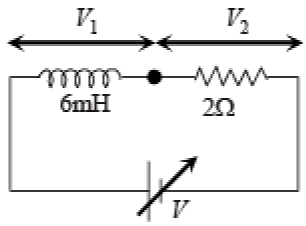
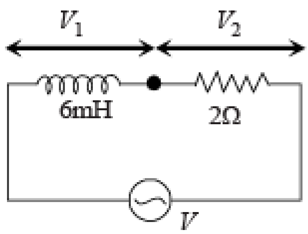
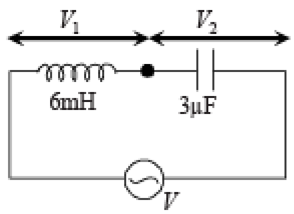
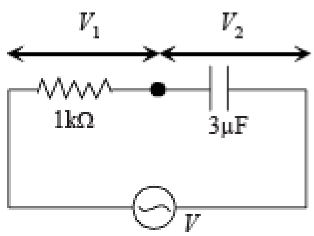
The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 , (indicated in circuits) are related as shown in Column I. Match the two

Column I	Column II
(A) $I \neq 0, V_1$ is proportional to I	(p) 

(B) $I \neq 0$, $V_2 > V_1$	(q) 
(C) $V_1 = 0, V_2 = V$	(r) 
(D) $I \neq 0, V_2$ is proportional to I	(s) 
	(t) 

Ans. A-r, s, t; B-q, r, s, t; C-p, q; D-q, r, s, t

Solution. The following are the important concepts which are applied in the given situation.

(i) For DC circuit, in steady state, the current I through the capacitor is zero. In case of L-C circuit, the potential difference across the inductor is zero and that across the capacitor is equal to the applied potential difference.

In case of L-R circuit, the potential difference across inductor is zero across resistor is equal to the applied voltage.

(ii) For AC circuit in steady state, I_{rms} curr ent flows through the capacitor, inductor and resistor. The potential difference across resistor, inductor and capacitor is proportional to I.

(iii) For DC circuit, for changing current, the potential difference across inductor, capacitor or resistor is proportional to the current.

Integer Value Correct Type

Q.1. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$**,**

the time constant (in millisecond) of the circuit is (2011)

Ans. 4

Solution. Time constant = RC

$$\text{Impedance} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\text{Given impedance} = R\sqrt{1.25}$$

$$\therefore\ R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\therefore\ RC = \frac{2}{\omega} = \frac{2}{500} \times 1000\ \text{ms}$$

$$\therefore\ RC = 4\ \text{ms}$$

Q.2. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a(a<<R) having two turns is placed with its centre at $z = \sqrt{3}R$ **along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by** $\frac{\mu_0 a^2}{2^{p/2} R}$**, then the value of p is (2012)**

Ans. 7

Solution. The magnetic field due to current carrying wire at the location of square loop is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi i R^2}{(R^2+3R^2)^{3/2}} = \frac{\mu_0 i}{16R}$$

The mutual induction

$$M = \frac{N\phi}{i} = \frac{2}{i}\left[\frac{\mu_0 i}{16R}\times a^2 \cos 45^\circ\right]$$

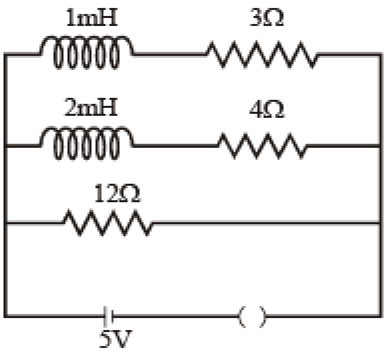
$$\therefore \quad M = \frac{\mu_0 a^2}{2^{\frac{7}{2}} R}$$

Q.3. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0.

The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is (JEE Adv. 2016)

Ans. 8

Solution.



$$\text{At } t = 0 \quad I_{min} = \frac{5}{12}$$

$$\text{At } t = \infty \quad I_{max} = \frac{5}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3}$$

$$\left[\frac{1}{R_{eq}}=\frac{1}{3}+\frac{1}{4}+\frac{1}{12}=\frac{8}{12}\right]$$

$$\therefore \frac{I_{\max}}{I_{\min}}=\frac{10}{3}\times\frac{12}{5}=8$$